

LECTURE NOTES 2-1: THE TANGENT AND VELOCITY PROBLEMS

The importance of a good question.

QUESTION 1: Given the graph of a function $y = f(x)$ and a point P on this graph, how do you *define* and *find* the equation of the tangent line to the graph at P ?

QUESTION 2: Given the position of an object (say a cell phone) at any time, how do you *define* and *find* the velocity of the object at a particular instant (say the moment your child launches it off a cliff)?

Some Facts:

- These questions are old. (200BC or older depending on your interpretation)
- These questions are hard, taking more than a thousand years and untold numbers of mathematicians to answer.
- Before finding solid mathematical ground, some of its ideas were even more controversial than Donald Trump's tweets are today!
- Attempts to answer these two questions is *part* of what led to the development/discovery of Calculus.
- The ideas you learn in calculus explain planetary motion or where a projectile will land or predict how fast an infection will spread.
- **Most importantly and perhaps obviously**, *the questions that motivated the development of calculus go a long way to explaining the definitions and applications we see later*

Example 1: Let $f(x) = (10 - x^2)/2$.

1. Sketch a LARGE graph of $f(x)$ in the space to the right. Include any x - or y -intercepts.
2. Let P be the point on the curve where $x = 1$ and let Q be the point on the curve where $x = 3$. Find the y -coordinate for P and Q and plot them on your graph above.
3. DEFINITION: A *secant line* on a graph is simply the line determined by two points on the graph. Find the EQUATION of the secant line determined by the points P and Q and graph it above.
4. Label the line you just plotted above with its *slope*.
5. For the FIVE points Q_1, Q_2, Q_3, Q_4, Q_5 with x -coordinates 2, 1.5, 1.25, 1.125, 1.0625, find the y -coordinate, plot the point, plot the secant line determined by P and Q_i , and label the line with its slope.

6. Sketch what YOU think the tangent to $f(x)$ at the point P should look like...???
7. What do you observe about the relationship between the secant lines you **calculated** and the tangent line you **guessed at**?

8. What is the significance of the **words in bold** in the previous question?

9. What PART of the tangent line is indicated by the sequence of secant lines? _____
10. Write the *equation* of the tangent line to $f(x)$ at P . Does this answer seem reasonable? Why or why not?

11. In *plain old ENGLISH SENTENCES* how would you explain (step-by-step) how to find the *equation* of the tangent line?

12. In the previous exercise, we chose points (Q_i 's) on the *right* of the point P , what would happen if we had chosen points on the *left*?

Example 2

t (seconds)	0	1	2	3	4	5	6
s (feet)	0	4.9	20.6	46.5	79.2	124.8	176.7

 The table above shows the position of a motorcyclist after accelerating from rest.

1. Find the average velocity for each time period and include units in your answer.

(a) From $t = 2$ to $t = 4$.

(b) From $t = 3$ to $t = 4$.

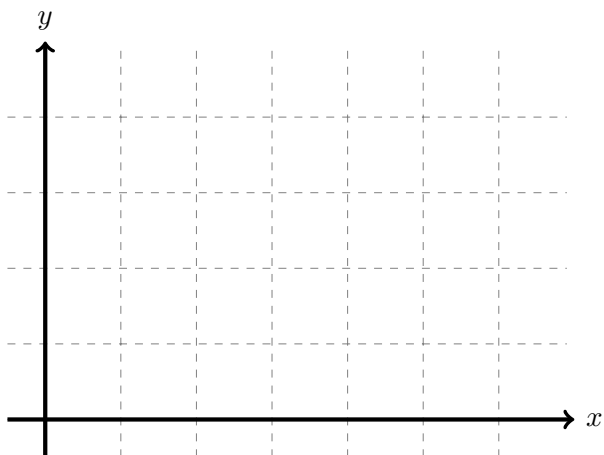
(c) From $t = 4$ to $t = 5$.

(d) From $t = 4$ to $t = 6$.

2. In words in English, what should the *average velocity* of an object be? *instantaneous velocity*?

3. If the object is a motorcycle, what should the *instantaneous velocity* of an object be?

4. Make a reasonable guess at the graph of s as a function of t using the data from the table.



5. Estimate the instantaneous velocity of the motorcycle four seconds after accelerating from rest. Is your answer reasonable?

6. On the graph in part 5, DRAW and LABEL all of the calculations from parts a-d in question 1.
7. State explicitly and in complete sentences in English the relationship between the tangent problem and the velocity problem.

The Last Word(s): In real estate, the three most important things to keep in mind are (1) location, (2) location, (3) location.

As of today, you now know the three most important techniques in calculus: